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Ballistic charge-carrier kinetics and phonon drag in semiconducting point contacts

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Abstract. New types of thermoelectric effects occurring in semiconducting point contacts whose dimensions are smaller than the inelastic electron and phonon relaxation lengths are analysed. Among them are the reduction of thermoelectric electromotive force in the point contact relative to the bulk material, making possible the determination of the phonon-drag contribution to the Seebeck coefficient, the heat release asymmetry in the banks of the contact, and hot-spot formation in the area of current concentration. These effects are of interest in an investigation of relaxation mechanisms in semiconductors as well as in studying possible new phenomena occurring in high-level integration electronic microcircuits.

1. Introduction

The tendency to miniaturization of electronic devices, which has stimulated progress in microelectronics, has led to the advent of very-large-scale integration devices whose elements have linear dimensions approaching 10^{-5} cm. If such microcircuits representing various types of electric contacts operate at liquid-nitrogen temperature ($T < 100$ K), their geometric dimensions can become comparable to or even smaller than characteristic electron and phonon scattering lengths in semiconductors on which the majority of microcircuit elements are based. The thermoelectric effects arising in such contacts, i.e point contacts between the non-degenerate semiconductors, are the object of our study.

The kinetic phenomena are drastically modified in point contacts as compared to bulk conductors. Since the applied voltage gradient is concentrated in the vicinity of the constriction (at a distance of the order of the contact diameter d, which is supposed to be smaller than the inelastic mean free path of the electron, l_e), it results in the appearance of a non-equilibrium distribution of electrons. In metallic contacts, the analysis of corresponding non-linearities in the current–voltage characteristic (on which point-contact spectroscopy [1–4] is based) permits an investigation of quasi-particle excitations (e.g. phonons interacting with electrons). The same method of point-contact spectroscopy has also recently been extended to degenerate semiconductors [5, 6]. Another area of non-equilibrium phenomena in point contacts is opened on applying a temperature difference, $\Delta T = T_2 - T_1$, to the contact. Thermoelectric effects in point contacts are peculiar due to a strong non-equilibrium distribution of phonons [7] interacting with non-equilibrium electrons.

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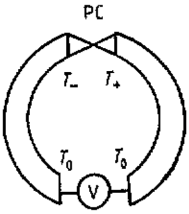

Figure 1. Thermoelectric circuit incorporating point contact (PC). Both sides of the contact are of similar material but nevertheless a thermoelectric voltage appears in the circuit.

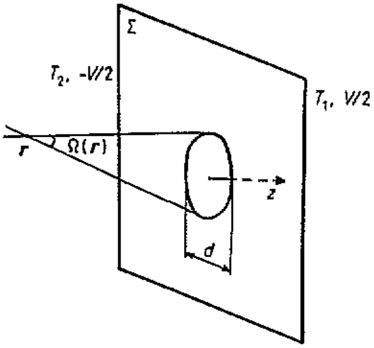

Figure 2. The model of the contact as 'an orifice in a screen'. Σ is a thin sheet impenetrable to electrons and phonons. $\Omega(r)$ is the solid angle at which the orifice is seen from a given point r.

Non-equilibrium phonons dramatically influence the Seebeck and Peltier effects in point contacts. According to the theory of thermoelectric effects developed in [8, 9], the diffusion (i.e. electronic) contribution to the thermopower at low temperature is the same as in the bulk metal, whereas the phonon-drag contribution to the contact thermopower is suppressed. In a contact, the latter contains an extra small factor $d/l_{\text{ph-e}}$ compared to the bulk, where $l_{\text{ph-e}}$ is the phonon–electron relaxation length. As has been proved in [8, 9], this permits one to measure the absolute phonon-drag thermopower of a metal by use of a circuit incorporating a point contact between similar metallic electrodes (figure 1). Measurement of the Seebeck coefficient in metallic point contacts [10] appeared to be in reasonable agreement with the above theory [8, 9].

Along with the Seebeck effect, the homogeneous metallic configuration incorporating point contact displays the Peltier effect manifesting itself in asymmetric heat release in the contact banks [9]. The latter effect has also been observed experimentally [11].

Experimental investigations of point-contact thermoelectric effects in conductors other than metals have been made using contacts based on a non-degenerate semiconductor, e.g. n-type Si, p-type Si and n-type GaAs [12–14]. Previously some interesting thermoelectric investigations of another type of semiconducting microstructure were reported [15–17]. Semiconducting contacts require specific theoretical consideration as many approximations valid for metals are not justified there; e.g. the electronic mean free path is energy-dependent, which influences contact thermopower; in some cases charge neutrality and classical approximation ($d \gg \lambda$, where λ is the de Broglie wavelength) are violated, etc.

This paper presents a theory accounting for the thermoelectricity in semiconducting point contacts. The general equations describing the kinetics of the contact are given in section 2. In section 3, thermoelectric coefficients are calculated in the ballistic limit, $l_i \gg d$, where l_i is the electron–impurity elastic scattering length. Unlike in metallic contacts, the electronic contribution to the thermopower differs from the appropriate value corresponding to the bulk semiconductor. In the diffusive regime ($l_i \ll d$, these quantities are equal, and therefore in the circuit of figure 1 they cancel each other. Section 4 is devoted to the analysis of the phonon-drag thermopower in semiconducting point contacts. This contribution is suppressed, with respect to the bulk, in the ballistic case $d \ll l_{\text{ph-e}}$. The effect of an asymmetric heat release in semiconducting point contacts is analysed in section 5. As is shown in section 6, the heat asymmetry decreases as the hot-carriers regime in the contact sets in. The final section deals with the thermoelectric effects in two-dimensional point contacts.

2. Basic equations

The nature of thermoelectric phenomena in point contacts of non-degenerate semiconductors depends upon the relation between the contact dimension and the characteristic electron and phonon scattering lengths. We assume that the contact diameter d is large compared to the de Broglie wavelength λ_{B} and the Debye screening radius r_{D}, and, on the other hand, that d is small compared to the inelastic electron and phonon relaxation lengths l_{e} and l_{ph}:

$$\lambda_{\text{B}}, r_{\text{D}} \ll d \ll l_{\text{e}}, l_{\text{ph}}. \tag{1}$$

With these conditions obeyed, we can neglect surface phenomena in semiconductors, e.g. band bending.

The inelastic electron relaxation length l_{e} depends upon the electric field in the contact, i.e. is determined by the current-carrying regime. It coincides with the electron scattering length $l_{\text{e-ph}}$ in the ballistic regime ($d \ll l_{\text{i}}$), and equals $(l_{\text{i}} l_{\text{e-ph}})^{1/2}$ in the diffusive regime ($d \gg l_{\text{i}}$). Note that at $T \sim 100$ K and at the carrier concentration $n \sim 10^{17}\ \text{cm}^{-3}$, the condition (1) is obeyed for contact diameters $d \sim 10^{-5}$ cm.

An orifice of diameter d in a screen impenetrable to electrons and phonons (see figure 2) is commonly used as a model of the point contact [4]. The bulk banks of the contact to which the current leads are attached and potential difference V is applied are assumed to be maintained at different temperatures, T_1 and T_2, respectively. The specific feature of the kinetic regimes considered is the appearance of strongly non-equilibrium electrons and phonons with different effective temperatures.

The kinetic equations in the contact include an equation for the electron distribution function f_p allowing for both elastic and inelastic scattering

$$\boldsymbol{v}\ \partial f_p/\partial \boldsymbol{r} + e\boldsymbol{E}\ \partial f_p/\partial \boldsymbol{p} - I_{\text{i}}(f_p) = I_{\text{e-ph}}(f_p, N_q^\alpha) \tag{2}$$

and an equation for the phonon distribution function N_q^α

$$\boldsymbol{u}^\alpha\ \partial N_q^\alpha/\partial \boldsymbol{r} = 0. \tag{3}$$

Here $\boldsymbol{v} = \partial\varepsilon/\partial\boldsymbol{p}$ and $\boldsymbol{u}^\alpha = \hbar\ \partial\omega^\alpha/\partial\boldsymbol{q}$ are the electron and phonon drift velocities, respectively. The electron–impurity $I_{\text{i}}(\ldots)$ and electron–phonon $I_{\text{ph}}(\ldots)$ collision integrals in equation (2) are of standard form. The electric field equals $\boldsymbol{E} = -\nabla\Phi(\boldsymbol{r})$, where Φ obeys the boundary condition $\Phi(r \to \infty) = (V/2)\operatorname{sgn} z$, and V is the voltage applied to the contact. $\Phi(\boldsymbol{r})$ is found from the electrical neutrality condition.

Far from the contact, the electron and phonon distribution functions approach their equilibrium values

$$f_p(r \to \infty, z > 0) = n_{\text{B}}[(\varepsilon_p - \mu_1)/T_1] \qquad f_p(r \to \infty, z < 0) = n_{\text{B}}[(\varepsilon_p - \mu_2)/T_2] \tag{4}$$

$$N_q^\alpha(r \to \infty, z > 0) = n_{\text{P}}(\hbar\omega_q^\alpha/T_1) \qquad N_p^\alpha(r \to \infty, z < 0) = n_{\text{P}}(\hbar\omega_q^\alpha/T_2) \tag{5}$$

where $n_{\text{B}}(x) = \exp(-x)$ and $n_{\text{P}}(x) = 1/[\exp(x) - 1]$ are the equilibrium Boltzmann and

Planck functions, respectively. The temperature dependence of the chemical potential μ in a doped semiconductor is (e.g. see [18])

$$\mu = T \ln[4\pi^3\hbar^3 n/(2\pi m T)^{3/2}]. \tag{6}$$

The electric current I and the entropy flux Π are calculated according to the relations

$$I = \int \mathrm{d}^2\rho\, j_z|_{z=0} \qquad \Pi = \int \mathrm{d}^2\rho\, \pi_z|_{z=0} \tag{7}$$

where integrals are taken over the orifice of the contact. Charge and entropy flux densities are

$$\boldsymbol{j} = 2e(2\pi\hbar)^{-3} \int \mathrm{d}^3p\, \boldsymbol{v}(\boldsymbol{p}) f_p \tag{8}$$

$$\boldsymbol{\pi} = 2(2\pi\hbar)^{-3} \int \mathrm{d}^3p\, \boldsymbol{v}(\boldsymbol{p})(1 - \ln f_p) f_p. \tag{9}$$

3. Thermoelectric coefficients of a point contact in the elastic scattering approximation

We start with a consideration of elastic scattering. Assuming that $I_{\text{e-ph}} = 0$ in equation (2), the electron and phonon distributions can be represented as

$$\begin{aligned} f_p^{(0)}(\boldsymbol{r}) = {} & \alpha_p(\boldsymbol{r}) n_{\mathrm{B}}((\varepsilon_p + e\Phi - eV/2 - \mu_1)/T_1) \\ & + [1 - \alpha_p(\boldsymbol{r})] n_{\mathrm{B}}((\varepsilon_p + e\Phi + eV/2 - \mu_2)/T_2) \end{aligned} \tag{10}$$

$$n_q^{\alpha(0)}(\boldsymbol{r}) = \beta_{u^{\alpha}(q)}(\boldsymbol{r}) n_{\mathrm{P}}(\hbar\omega_q^{\alpha}/T_1) + [1 - \beta_{u^{\alpha}(q)}(\boldsymbol{r})] n_{\mathrm{P}}(\hbar\omega_q^{\alpha}/T_2) \tag{11}$$

where $\alpha_p(\boldsymbol{r})\,(\beta_{u(q)}(\boldsymbol{r}))$ are, similarly to [9], the probabilities that an electron (phonon) with momentum $\boldsymbol{p}\,(\boldsymbol{q})$ arrives at the given point $\boldsymbol{r}$ from the right-hand side of the contact. We further assume that the following condition holds

$$|eV|, |\Delta T| \ll T_{1,2}, |\mu_{1,2}| \tag{12}$$

at which the electron and phonon trajectories can be considered as straight lines. In the ballistic regime, we have $\alpha_p(\boldsymbol{r}) = \beta_u(\boldsymbol{r}) = 1$ for the velocity $\boldsymbol{v}(\boldsymbol{u})$ within the solid angle $\Omega(\boldsymbol{r})$ at which the contact orifice is seen from the point $\boldsymbol{r}$ (figure 2), whereas for the rest of directions these quantities are equal to zero, $\alpha_p = \beta_u = 0$.

Taking into consideration equation (12), we obtain in the case of arbitrary impurity concentration an equation for the function $\alpha(\boldsymbol{r})$

$$\boldsymbol{v}\, \partial\alpha_p(\boldsymbol{r})/\partial\boldsymbol{r} = I_{\mathrm{i}}(\alpha_p) \tag{13}$$

with the boundary condition $\alpha_p(r \to \infty) = \Theta(z)$. A similar equation holds for the function $\beta_u(\boldsymbol{r})$.

In the diffusive regime, equation (13) is solved to give

$$\alpha_p(\boldsymbol{r}) = \alpha_0(\boldsymbol{r}) - l_{\mathrm{i}}(\boldsymbol{p}/p)\, \partial\alpha_0/\partial\boldsymbol{r} \tag{14}$$

where

$$\alpha_0(\boldsymbol{r}) = \Theta(z) - \varphi_0(\boldsymbol{r}) \operatorname{sgn} z$$

and

$$\varphi_0 = (1/\pi) \tan^{-1}\{2r^2d^{-2} - \tfrac{1}{2} + 2[(r^2d^{-2} - \tfrac{1}{4})^2 + z^2d^{-2}]^{1/2}\}^{-1/2}. \tag{15}$$

In the linear approximation in V, ΔT, the expressions for the elastic components of the current $I^{(0)}$ and the electron heat flow $Q^{(0)}$ can be written as

$$I^{(0)} = -(V^*/R^{\mathrm{V}}) + K\Delta T \tag{16}$$

$$Q^{(0)} = -TKV^* + \Delta T/R^{\mathrm{T}} \tag{17}$$

where $V^* = V + (\mu_1 - \mu_2)/e$. R^{V} is the electrical resistance of the contact. The value of R^{T} is related to the thermal resistance R^{T} according to the formula

$$(R'^{\mathrm{T}})^{-1} = (R^{\mathrm{T}})^{-1} - TR^{\mathrm{V}}K^2.$$

Note that at small ΔT the relation $Q^{(0)} = T\Pi^{(0)}$ holds. The fact that the expressions for $I^{(0)}$ and $Q^{(0)}$ involve the same thermoelectric coefficient K follows from Onsager's principle.

The kinetic coefficients describing the current and heat flow have different forms in the ballistic and diffusive regimes. In the ballistic case ($l_{\mathrm{i}} \gg d$) we have

$$R^{\mathrm{V}}_{\mathrm{B}} = (2\pi^2\hbar^3/e^2mST)\exp(-\mu/T) \tag{18}$$

$$R^{\mathrm{T}}_{\mathrm{B}} = (\pi^2\hbar^3/mST^2)\exp(-\mu/T) \tag{19}$$

$$K = (emST/2\pi^2\hbar^3)(2 - \mu/T)\exp(-\mu/T) \tag{20}$$

where $S = \pi d^2/4$ is the contact area. The relation $R^{\mathrm{V}}/R^{\mathrm{T}} = 2T/e^2$ ensures the applicability of the Wiedemann–Franz law for the relation between electrical and thermal resistances of the contact. Besides, the substitution of the chemical potential (7) into equations (18) and (19) supports the validity of Sharvin's formula for a semiconducting contact (the latter says $R^{\mathrm{V}}_{\mathrm{B}} \sim l/\sigma S$ and $R^{\mathrm{T}}_{\mathrm{B}} \sim l/\kappa S$, where σ and κ are the electrical and thermal conductivities, respectively).

The electronic component of the thermoelectric power for a point contact equals

$$S^{\mathrm{e}}_{\mathrm{B}} = (2 - \mu/T)/e. \tag{21}$$

In the diffusion regime ($l_{\mathrm{i}} \ll d$), the kinetic coefficients of the contact are expressed as

$$R^{\mathrm{V}}_{\mathrm{D}} = \frac{3\pi^2\hbar^3e^{-2}}{2(2m)^{1/2}A_rT^{r+3/2}\Gamma(r+\frac{5}{2})d}\exp(-\mu/T) \tag{22}$$

$$R^{\mathrm{T}}_{\mathrm{D}} = [e^2/(r+\tfrac{5}{2})T]R^{\mathrm{V}}_{\mathrm{D}} \tag{23}$$

$$K_{\mathrm{D}} = [(r+\tfrac{5}{2}-\mu/T)/e^2]R^{\mathrm{V}}_{\mathrm{D}} \tag{24}$$

$$S^{\mathrm{e}}_{\mathrm{D}} = (r+\tfrac{5}{2}-\mu/T)/e \tag{25}$$

where r determines the energy dependence of the elastic electron relaxation time according to $\tau_r = A_r\varepsilon^r$, and $\Gamma(x)$ is Euler's gamma function. We have taken into consideration that $\int \mathrm{d}^2\rho\,(\mathrm{d}\alpha_0/\mathrm{d}z) = d$.

Note that the Wiedemann–Franz law is also obeyed in the diffusive regime. The substitution of the appropriate relaxation time into equation (22) leads to the Maxwell formula, $R^{\mathrm{V}} = 1/\sigma d$.

The thermopower in the diffusive regime coincides with that in the ballistic regime at the value of relaxation time exponent $r = -\frac{1}{2}$, which corresponds to the energy-independent electron mean free path. This situation is realizable in metals whereas in doped semiconductors electron scattering by ionized impurities dominates, which corresponds to $r = \frac{3}{2}$ [18].

The thermopower obtained from equations (21) and (25) should be compared with that for the bulk semiconductor $S_{\mathrm{m}}^{\mathrm{e}} = (r + \frac{5}{2} + \mu/T)/e$ [18]. The contact thermopower coincides with the latter quantity in the diffusive regime under the condition that elastic scattering is dominant.

In the ballistic limit, the coincidence of thermopower in the contact and in the bulk [8] is realizable when scattering of electrons by acoustic phonons ($r = -\frac{1}{2}$), in the bulk, dominates [18]. Therefore, the study of thermoelectric effects in point contacts provides information on the mechanisms of electron scattering.

4. Phonon-drag effects

The small size of a point contact compared to the inelastic electron scattering length ($d \ll l_{\mathrm{e}}$) permits us to find the inelastic correction $f_p^{(1)}$ to the electron distribution function $f_p^{(0)}$ (in the case of elastic electron scattering) by perturbatively considering the electron–phonon collision integral $I_{\mathrm{e\text{-}ph}}$. In what follows our interest will be concentrated on the inelastic component $I^{(1)}$ of the point-contact current, which will be calculated with the help of a technique originally developed for metallic point contacts and described elsewhere [19]. With arbitrary (Fermi or Boltzmann) statistics for conduction electrons, $I^{(1)}$ can expressed in the form

$$I^{(1)} = \frac{2e}{(2\pi\hbar)^3} \int \mathrm{d}^3 r \int \mathrm{d}^3 p \, [\alpha_{-p}(r) - \Theta(z)] \, I_{\mathrm{e\text{-}ph}}(f_p^{(0)}) \tag{26}$$

where we have taken into consideration the condition that the energy $\hbar\omega_{p-p'} \sim u(m\varepsilon_p)^{1/2}$ of a phonon participating in electron scattering is small compared to the electron energy $\varepsilon_p \sim T$. The dimensionless small parameter corresponding to this inequality is $(u^2 m/T)^{1/2} = (T_{\mathrm{S}}/T)^{1/2} \ll 1$, where $T_{\mathrm{S}} = mu^2 \sim 1$ K. The value of $I^{(1)}$ is calculated in the small-voltage limit $|eV| \ll T$, which corresponds to the thermal EMF calculated at $\Delta T \ll T$. This permits us to neglect the trajectory bending, in the calculation of probability factor $\alpha_p(r)$ entering equation (26). Considering the case $T \gg T_{\mathrm{S}}$ and using the expression $W_p^{\alpha} = \pi\hbar^2 p^2/9aMm\omega_p^{\alpha}$ [18] for the square of the matrix element of the electron–phonon interaction, we obtain in the ballistic regime

$$\begin{aligned} I_{\mathrm{B}}(V, \Delta T) = {} & I_{\mathrm{B}}^{(0)}(V^*, \Delta T)[1 - (4/3\pi)(d/l_{\mathrm{e\text{-}ac}})] \\ & - 0.66 I_{\mathrm{B}}^{(0)}(V, 0)(\Delta T/eV)(T_{\mathrm{S}}/T)^{1/2}(d/l_{\mathrm{e\text{-}ac}}). \end{aligned} \tag{27}$$

The quantity $I_{\mathrm{B}}^{(0)}$ can be found from equations (16), (18) and (20); $l_{\mathrm{e\text{-}ac}}$ is the length of quasi-elastic electron scattering by acoustic phonons, $l_{\mathrm{e\text{-}ph}} \sim 1/W$:

$$l_{\mathrm{e\text{-}ac}} = (9\pi/4)(\hbar^4 M u^2/a^3 m^2 T E_{\mathrm{a}}^2) \qquad E_{\mathrm{a}} = (\hbar^2/2ma^2).$$

In the diffusive regime ($l_{\mathrm{i}} \ll d$), the inelastic contribution to the current is almost independent of the parameters of elastic electron scattering. We have in this case

$$I_{\mathrm{D}}(V, \Delta T) = I_{\mathrm{D}}^{(0)}(V^*, \Delta T)(1 - d/l_{\mathrm{e\text{-}ac}}) + (\Delta T/eR_{\mathrm{B}}^{\mathrm{V}})(T_{\mathrm{S}}/T)^{1/2}(d/l_{\mathrm{e\text{-}ac}})c_{\mathrm{D}} \tag{28}$$

where $c_{\mathrm{D}} \sim 1$. This expression permits estimation of the phonon-drag thermopower in the contact.

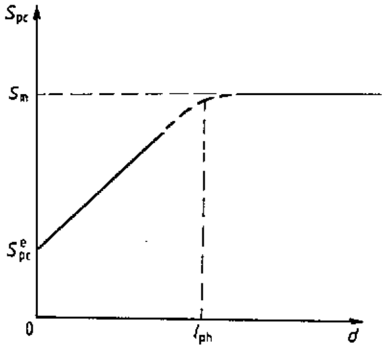

Figure 3. Dependence of thermoelectric coefficient of the point contact S_{pc} upon its diameter d (schematic). S_m is the thermo-EMF coefficient for a bulk semiconductor, and l_{ph} is the phonon mean free path.

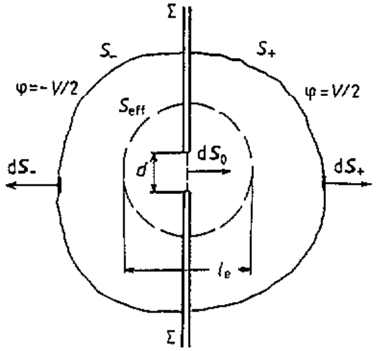

Figure 4. Schematic of the heat transfer in a point contact. S_{eff} is the effective boundary of the inelastic relaxation zone.

In the ballistic and diffusive regimes, equations (27) and (28) give us

$$S_B^{ph} = 0.66(T_S/T)^{1/2} d/el_{e\text{-}ph} \tag{29}$$

and

$$S_D^{ph} = c_D(r)(T_S/T)^{1/2} d/el_{e\text{-}ph} \tag{30}$$

respectively. The role of scattering mechanisms shows up as a variation of the coefficient c_D of the order of unity.

In bulk semiconductors, taking into account both phonon–electron and other mechanisms of phonon scattering characterized by relaxation lengths $l_{ph\text{-}e}$ and l_{ph}^*, the phonon-drag thermopower can be expressed as (e.g. see [18])

$$S_m^{ph} = (T_S/T)(\tau_{ph}/e\tau_{e\text{-}ph}) = (l_{ph}/el_{e\text{-}ph})(T_S/T)^{1/2} \tag{31}$$

where $\tau_{ph} = l_{ph}/u$ and $l_{ph}^{-1} = l_{ph\text{-}e}^{-1} + l_{ph}^{*-1}$ is the total phonon scattering length.

Equations (29) and (30) differ from the corresponding expressions for the bulk in the factor d/l_{ph}

$$S_{pc}^{ph} \simeq (d/l_{ph})S_m^{ph} \tag{32}$$

where ‘pc’ stands for the point-contact and ‘m’ for the bulk. For phonon–electron scattering alone

$$S_{pc}^{ph} \simeq (d/l_{ph\text{-}e})S_m^{ph}. \tag{33}$$

Equations (32) and (33) suggest that, in the semiconducting point contact, the phonon-drag contribution to the thermopower is suppressed compared to that in the bulk. Similar effects have been observed in metallic point contacts [10].

The difference between the thermopower in point contacts and in bulk conductors results in the appearance of a non-zero Seebeck voltage in a homogeneous semiconducting circuit incorporating a point contact (figure 1). The linear dependence of the differential phonon-drag thermopower S_{pc}^{ph} on the contact diameter d persists until the inequality $d \ll l_{ph}$ holds. With a further increase of the contact diameter, S_{pc}^{ph} saturates at the value corresponding to the phonon-drag thermopower of a bulk semiconductor. The dependence of S_{pc}^{ph} on the contact diameter is shown schematically in figure 3. The size effect in the phonon drag of a semiconducting point contact has been observed experimentally [13, 14]. Note the difference between this effect and the known size reduction of phonon drag in wires due to surface phonon scattering [20].

5. Heat production asymmetry in point contacts

In studying the heat production in semiconducting point contacts, Trzcinski *et al* [13, 14] found that the heat flow Q_+ released in the contact bank corresponding to more energetic electrons exceeds the heat flow Q_- of the bank whose electrons are slowed by the electric field. At the same time, the total heat production in the contact $Q = Q_+ + Q_-$ is described by the standard expression $Q = IR$. The point-contact heat flow asymmetry Q_a is determined by

$$Q_a = Q_+ - Q_- . \tag{34}$$

According to the results obtained for silicon point contacts [13], the ratio Q_a/Q may be as high as 0.3. A similar effect has been observed in a copper point contact [11]. The general theory of thermoelectric phenomena in point contacts taking into consideration both electron and phonon heat transport has been developed in [9].

For a non-degenerate electron gas, the heat production asymmetry in the point contact has two components: the electronic one, Q_a^{el}, resulting from the electron acceleration in the electric field, and the phonon part, Q_a^{ph}, corresponding to the anisotropic generation of non-equilibrium phonons.

For metals, Q_a^{el} is estimated as $Q_a^{el} \sim (eV/\varepsilon_F)Q \ll Q$, i.e. much less than the total heat dissipation, IR. In a low-density electron gas, the condition $|eV| \gg |\mu|, T$ leads to predomination of the electronic mechanism of asymmetry. Our aim is to calculate Q_a^{el} for a semiconducting point contact.

The heat production rates $Q_\pm$ in the bulk banks of a point contact can be calculated as heat flows through the surfaces S_+ and S_- embracing the contact regions in which inelastic electron and phonon relaxation take place (figure 4). If the surfaces S_+ and S_- are moved off the contact up to distances R exceeding the inelastic relaxation lengths $R \gg l_e, l_{ph}$, the heat flows Q_+ and Q_- become independent of the shape of S_+ and S_-. Since the drop of the potential applied to the contact occurs at characteristic distances of the order of the contact diameter d, the potential $\Phi(r)$ will reach its limiting values $\Phi_\pm = \pm V/2$ at the surfaces $S_\pm$.

In the bulk banks of the contact, electrons and phonons are almost at equilibrium. Therefore, the heat flow can be calculated using thermodynamic considerations. Taking into consideration the above values of the potential Φ at $S_\pm$, we obtain

$$Q_+ = \int \mathrm{d}S_+ \, W - I\mu(T_1)/e - IV/2 \tag{35a}$$

$$Q_- = \int \mathrm{d}S_- \, W + I\mu(T_2)/e - IV/2. \tag{35b}$$

Here, $W(r)$ is the total energy flow density at a given point r. In the stationary situation considered here, energy density is time-independent and hence W obeys the condition

$$\operatorname{div} W = 0.$$

Assuming that the total energy flow is localized within the point-contact cross section, we obtain the following expression for $Q_\pm$:

$$Q_\pm = \pm \int \mathrm{d}S_0\, W_{\mathrm{pc}} \mp I\mu(T_{1,2})/e - IV/2 \tag{36}$$

where W_{pc} is the energy flux density at $z = 0$,

$$W_{\mathrm{pc}} = (2\pi\hbar)^{-3} \int \mathrm{d}^3p \left(2v(p)[\varepsilon_p + e\Phi(r)]f_p + \hbar \sum_\alpha u_p^\alpha \omega_p^\alpha N_p^\alpha\right). \tag{37}$$

The total heat production $\Sigma_\pm$ in the contact sides is found by taking into account the heat flow Q_{pc} through the point contact:

$$\Sigma_+ = Q_+ - Q_{\mathrm{pc}} \qquad \Sigma_- = Q_- + Q_{\mathrm{pc}}.$$

Taking into consideration only the electronic part of Q_{pc} (see equation (9)), we obtain

$$Q_{\mathrm{pc}} = \int \mathrm{d}S_0\, W_{\mathrm{pc}}^{\mathrm{el}} - I^{(0)}\mu/e - (I^{(0)}V/2)\coth(eV/2T).$$

The relative total heat production asymmetry is

$$A = (\Sigma_+ - \Sigma_-)/(\Sigma_+ + \Sigma_-) = L(|e|V/2T) \tag{38}$$

where $L(x) = \coth(x) - 1/x$ is the Langevin function. At $|eV| \ll T$ we obtain $A = |e|V/6T$. If the condition $|eV| \ll T$ is violated, the total heat production asymmetry stops increasing linearly with V and saturates at $|eV| \gg T$, giving $A = \operatorname{sgn}(V)$.

At $|eV| \ll T$, the electronic component of the heat flow in the contact bank is of the same order of magnitude as the phonon part since

$$Q_{\mathrm{a}}^{\mathrm{el}}/Q_{\mathrm{a}}^{\mathrm{ph}} \sim (|eV|/T)(T_{\mathrm{S}}/T).$$

However, at $|eV| \gg T$ the value of the heat flow asymmetry Q_{a} is mainly determined by the electron contribution. In this case the relatively small temperature difference between the contact banks may be disregarded. Since for the non-degenerate semiconductors $\langle\varepsilon_p\rangle$, $|\mu| \sim T$, the condition $|eV| \gg T$ requires (unlike in previous cases) that in the calculation of the probability functions $\alpha_p(r)$ (see equation (11)) the trajectory distortion by the contact electric field should be taken into account.

Let $\Phi_{\mathrm{pc}}(r)$ denote the value of the potential at the orifice of the contact. Under the condition $\varepsilon_p + e\Phi_{\mathrm{pc}} < eV/2$ the electron is unable to reach the orifice from the right half-space. Therefore, in this case $\alpha_p(z = 0) = 0$. If, on the contrary, $\varepsilon_p + e\Phi_{\mathrm{pc}} < -eV/2$, the electron cannot come to the same point from the left half-space and therefore $\alpha_p(z = 0) = 1$. The general form of the function $\alpha_p(r)$ which follows from the above consideration is

$$\alpha_p(r, V) = \Theta[\varepsilon_p + e\Phi(r) - eV/2]\{1 - \varphi_p(r, V)\Theta[\varepsilon_p + e\Phi(r) + eV/2]\} \tag{39}$$

where $\Theta(x)$ is the Heaviside step function. The quantity $\varphi_p(r, V)$ in equation (39) falls within the limits $0 \leqslant \varphi \leqslant 1$. At $|eV| \gg T$ the calculation of the explicit form of this function is not necessary. As follows from equation (39), the non-zero contributions to the current and heat flow will correspond only to those regions of momentum space in which the condition $\varepsilon_p + e\Phi_{\mathrm{pc}} \geqslant eV/2$ is satisfied.

The general expression for the current, in the absence of inelastic scattering, is, according to equations (8), (10) and (39),

$$I^{(0)} = (e/2\pi^3\hbar^3 m)\exp(\mu/T)\sinh(eV/2T)F_1(V) \tag{40}$$

where

$$F_1(V) = \int \mathrm{d}S_0 \int \mathrm{d}^3p\, p_z \alpha_p(\boldsymbol{r}, V)\exp\{-[\varepsilon_p + e\Phi(\boldsymbol{r})]/T\}\,\Theta(\varepsilon_p + e\Phi^{(0)} - eV/2). \tag{41}$$

In the case $|eV| \ll T$ the quantity F_1 does not depend on V and equals $F_1 = -2\pi S(mT)^2$. A detailed analysis of the I–V characteristic and the calculation of the potential distribution $\Phi^{(0)}(\boldsymbol{r})$ in a semiconducting point contact is given in [21].

The electron component of the energy flow through the point contact can be presented in the form

$$\int \mathrm{d}S_0\, W_{\mathrm{pc}}^{\mathrm{el}} = (2\pi^2\hbar^3 m)^{-1}\exp(\mu/T)\sinh(eV/2T)F_2(V) \tag{42}$$

where

$$F_2(V) = \int \mathrm{d}S_0 \int \mathrm{d}^3p\, p_z(\varepsilon_p + e\Phi^{(0)})\alpha_p(\boldsymbol{r}, V) \times \exp\{-[\varepsilon_p + e\Phi^{(0)}(\boldsymbol{r})]/T\}\Theta(\varepsilon_p + e\Phi^{(0)} - eV/2). \tag{43}$$

The comparison between the dependences $F_1(V)$ and $F_2(V)$ in the limit $|eV| \gg T$ results in the relation

$$F_2(V) = (eV/2)F_1(V)[1 - O(T/eV)] \tag{44}$$

where $O(x) \to 0$ at $x \to 0$. According to equation (44), the maximal value of the energy flow through the orifice of the point contact is

$$\int \mathrm{d}S_0\, W_{\mathrm{pc}}^{\mathrm{el}} = \tfrac{1}{2}I^{(0)}V\,\mathrm{sgn}(eV). \tag{45}$$

Taking into account equation (37), we obtain the expression for the heat flow in the contact banks as ($|eV| \gg T$)

$$Q_\pm = -\tfrac{1}{2}I^{(0)}V[1 \mp \mathrm{sgn}(eV)] \mp I^{(0)}\mu/e. \tag{46}$$

Taking into consideration the condition $|eV| \gg |\mu|$, which is valid at high voltages, and using equation (46), we arrive at the conclusion that the limit value of heat flow asymmetry can be as high as unity:

$$Q_a/Q = (Q_+ - Q_-)/(Q_+ + Q_-) = -\mathrm{sgn}(eV). \tag{47}$$

Note that according to equations (44) and (47) the limit heat flow asymmetry is independent of the mechanism of elastic electron scattering as the impurity concentration does not enter into the expression (46). However, this is justified only in the case of weak inelastic scattering ($d \ll l_e$). In a dirty contact ($l_i \ll d$), the condition $l_e = (l_i l_{\text{e-ph}})^{1/2} < d$ holds, owing to which the heat release asymmetry amounts to the value given by equation (47). The reduction of the relative asymmetry Q_a/Q at increasing contact size was observed in [12, 14].

The temperatures of the contact banks $T_\pm$ depend on the heat flows $Q_\pm$ and on the conditions of heat removal from the banks. To estimate the bank temperature, we confine ourselves to the simplest model of the contact shown in figure 2.

The temperature can be measured at macroscopic distance R from the contact ($R \gg l_{\text{e-ph}}, l_{\text{ph}} \gg d$) at which the contact can be considered as a point heat source. The heat propagation in the bulk banks is described by the equations of thermal conductivity with a point source of heat

$$-\nabla\kappa\nabla T_\pm(r) = 2(Q_\pm \mp I\Pi_{\text{m}})\delta(r) \tag{48}$$

subject to the boundary condition corresponding to the absence of heat flow across the surface Σ of a contact

$$z\nabla T_\pm/\Sigma = 0.$$

Here, κ and $\Pi_{\text{m}} = TS_{\text{m}}$ are the thermal conductivity and the Peltier coefficient of the bank materials, respectively. The small temperature deviation $\Delta T_\pm(r)$ from the equilibrium value T_0 corresponds to the conditions of the experiment.

Equation (48) is solved to give

$$T_\pm(r) = T_0 + (Q_\pm \mp I\Pi_{\text{m}})(2\kappa r)^{-1}. \tag{49}$$

Taking into account the condition $|eV| \gg T$ which results in the inequality $|IV| \gg |I\Pi_{\text{m}}|$ and using equation (46) we obtain

$$T_\pm(r) = T_0 + \tfrac{1}{2}|IV|[1 \mp \operatorname{sgn}(eV)](2\pi\kappa r)^{-1}. \tag{50}$$

As follows from the latter formula, at $|eV| \gg T$ the asymmetric part of the temperature distribution is proportional to V^2. This is typical for the case of elastic scattering. Taking an estimate of κ valid for non-degenerate semiconductors, $\kappa = nl_{\text{i}}(T/m)^{1/2}$, and using the expression for the point-contact current derived in section 3, we obtain in the ballistic regime

$$\Delta T_{\text{B}}/T_0 \simeq IV/T_0\kappa r \simeq (eV/T_0)^2(d^2/l_{\text{i}}r). \tag{51}$$

In the diffusive regime, the temperature difference decreases compared to (51) in proportion to l_{i}/d, which gives us

$$\Delta T_{\text{D}}/T_0 \simeq (eV/T_0)^2(d/r). \tag{52}$$

The temperature measured at macroscopic distances from the contact is only a percentage deviation from the ambient temperature T_0 [12, 14]. Note, however, that at the minimal distance at which the notion of temperature is sensible ($R \geqslant l_{\text{e-ph}}, l_{\text{ph}}$) the estimates following from equations (51) and (52) give a magnitude of ΔT as high as ambient temperature T_0.

6. Heat production asymmetry in the 'hot-spot' regime

In the metallic point contacts, the heat release asymmetry is negligible in the case of strong electron–phonon scattering ($l_{\text{e}} \ll d$), as has been discussed above [22]. Contrary to this, in a semiconducting contact the heat asymmetry is prominent, and at sufficiently large voltage the regime of hot electrons sets in. We suppose that the main source of electron scattering is that by the acoustic phonons ($l = l_{\text{ac}}$). Electron heating in this case corresponds to the electric field in the contact of the order of $E_0 = (6T_ST)^{1/2}(el_{ac})^{-1}$ [23]. At the electronic mean free path l_{ac} of the order of 10^{-6} cm (appropriate to room temperature) and T_S of the order of 1 K this gives an estimate $E_0 \sim 10^3\ \mathrm{V\,cm^{-1}}$. In experiments [12–14] contact diameter d was estimated as $d \sim 10l_{ac}$ at voltage $V \sim 10^2$ mV. This corresponds to the value of the electric field E in the contact area of order of $10^4\ \mathrm{V\,cm^{-1}}$. Therefore, the hot-spot does appear in the near vicinity of the contact.

The electronic energy flux in the constriction of the point contact can be estimated at $E \gg E_0$ as

$$W = I(V)(8/\pi)^{1/2}[TE(V)/eE_0]. \tag{53}$$

According to equation (37), the heat production asymmetry will be in this case

$$Y = (Q_+ - Q_-)/(Q_+ + Q_-) = W/|IV| = (8/\pi)^{1/2}(T/|eV|)[E(V)/E_0]. \tag{54}$$

In the approximation considered, the average kinetic energy of a hot electron equals $\langle\varepsilon\rangle = cTE/E_0$, resulting in the heat release asymmetry

$$Y = (8/\pi)^{1/2}\langle\varepsilon\rangle/c|IV| \qquad c \sim 1. \tag{55}$$

Substitution of the above expression into equation (54) gives

$$Y = 2(T/3\pi T_S)^{1/2} l_{ac}[E(V)/E_0]. \tag{56}$$

As the contact field is estimated as EV/d we find

$$Y \simeq (T/T_S)^{1/2} l_{ac}/d. \tag{57}$$

According to this equation the heat flow asymmetry, at small Knudsen number $K = (l_{ac}/d) \ll 1$, is proportional to K in accord with [12] and [14].

7. Ballistic kinetic phenomena in two-dimensional contacts

The analysis conducted so far has been concerned with the case when the geometrical dimension of the contact exceeded the de Broglie wavelength of the electron. Recent progress in the technology of heterojunction fabrication has resulted in the advent of two-dimensional point contacts of controlled geometry [24, 25]. These are shown schematically in figure 5. The two-dimensional nature of the current distribution in such contacts is responsible for their specific kinetic properties, which differ significantly from those corresponding to three-dimensional contacts. Confining our consideration to elastic scattering ($I_{e\text{-}ph}(\ldots) = 0$), we obtain the following expression for the current (in the case of degenerate electron statistics)

$$I^{(0)} = (e^2/\pi\hbar)(k_Fd/\pi)[V^* + (\pi^2/6e)(T\Delta T/\mu)]. \tag{58}$$

Here k_F is the Fermi wavevector. According to equation (58), the diffusive (electronic) component of the thermopower equals

$$S^e_{pc} = \pi^2 T/6e\mu. \tag{59}$$

Note a factor k_Fd increase in the two-dimensional point-contact resistance compared to the three-dimensional contact, and a numerical factor 2 decrease in the thermopower [9].

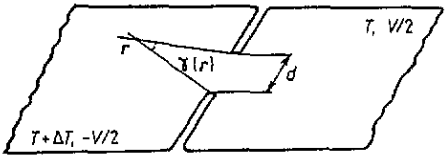

Figure 5. Two-dimensional point contact. $\gamma(\boldsymbol{r})$ is the angle at which the orifice is seen from the point $\boldsymbol{r}$.

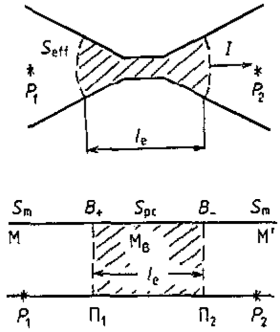

Figure 6. Equivalent scheme for the thermoelectric effects in the point contact. M and M' are the bulk banks, and M_B the 'ballistic element'. At the boundary B_+ of the ballistic zone the heat is released as a result of relaxation of hot electrons, whereas at the boundary B_- the heat is absorbed. The thermoelectric voltage in the circuit is induced due to the difference between the thermo-EMF of the ballistic element and that of the bulk.

The electric potential distribution in the contact is similar to that of [1]

$$\Phi(\boldsymbol{r}) = (V/2)[1 - \gamma(\boldsymbol{r})/\pi]\,\mathrm{sgn}\, z \tag{60}$$

where $\gamma(\boldsymbol{r})$ is the angle at which the orifice is seen from the point $\boldsymbol{r}$.

In the case of non-degenerate electron statistics, the expressions for the current, equation (58), and the thermopower, equation (59), of a two-dimensional point contact are modified as follows:

$$I^{(0)} = (ed/2\pi^2\hbar^3)(2\pi mT)^{1/2}\exp(\mu/T)[eV^* + \Delta T(\mu/T - \tfrac{3}{2})] \tag{61}$$

$$S^{\mathrm{c}}_{\mathrm{pc}} = (\tfrac{3}{2} - \mu/T)/e. \tag{62}$$

The difference in the kinetic coefficients of point contacts of lower dimensions suggests the existence of a transverse size effect setting in when the thickness becomes comparable to or less than the de Broglie wavelength of conduction electrons.

8. Conclusions

If the contact dimension d is smaller than the lengths of the inelastic electron and phonon scattering, the electron distribution in the contact transforms to a specific form that cannot be described as the conventional current-carrying drift state of $\boldsymbol{p}\boldsymbol{v}(\partial f_0/\partial\varepsilon)$ type. The electron distribution function $f(\boldsymbol{p}, \boldsymbol{r})$ should be calculated in this case according to the scheme proposed in section 3 introducing the probabilities of electron and phonon arrival near the contact from the contact banks. The electron–phonon interaction in the near vicinity of the contact described by the electron–phonon, $I_{\text{e-ph}}$, and phonon–electron, $I_{\text{ph-e}}$, collision integrals results in non-linear corrections to the kinetic coefficients.

Under the assumption of elastic electron scattering, the size effect of the contact is controlled by the parameter d/l_i, where l_i is the electron–impurity scattering length. In the diffusive regime ($d \gg l_i$), the electrical (R^V) and thermal (R^T) resistances as well as the electronic component of the thermopower (S^e_{pc}) coincide with those quantities appropriate for bulk semiconductors. In the ballistic regime corresponding to contact diameter smaller than the mean free path ($d \ll l_i$), the contact diameter itself plays the role of the scattering length, which results in Sharvin's formula for the resistance. The temperature dependence of the quantities R^V, R^T and S^e_{pc} is therefore determined by the energy-independent scattering length.

The allowance for the weak electron–phonon interaction in the contact results in an additive component of the resistance proportional to the small parameter $d/l_{e\text{-}ph}$. The point-contact spectroscopy of phonons in metals and degenerate semiconductors [1–6] is one of the consequences of this contribution, which is non-linear with respect to the voltage V applied to the contact. In a non-degenerate semiconductor, the inelastic component of the resistance provides less detailed information concerning the phonon spectrum since the energy distribution of conduction electrons has no sharp edge.

In a non-degenerate semiconductor, a pronounced size effect occurs at $d \ll l_{e\text{-}ph}$, $l_{ph\text{-}e}$. This involves a $d/l_{ph\text{-}e}$ times reduction in the phonon-drag thermopower of the point contact as compared to the bulk semiconductor. The above phenomena, including the difference between S^e_{pc} and S^e_m for point-contact and bulk material, lead to the non-zero thermoelectric voltage in the circuit made of single material but incorporating a point contact.

The heat release asymmetry in a homogeneous symmetrical point contact is another manifestation of the electron–phonon interaction. If the condition $|eV| \gg T, |\mu|$ is satisfied, the main part of the energy dissipated in the contact can be released in the contact bank with a higher potential.

Both phenomena have been observed experimentally in point contacts [12–14]. They do not have their counterparts in circuits made of bulk conductors. These phenomena, which can naturally be referred to as ballistic Seebeck and Peltier effects, can be explained within the scheme representing the non-equilibrium region of the point contact as the 'ballistic element' introduced in a circuit made of similar conductors (figure 6). Since the dimension of the near-contact region where the electric potential and/or temperature gradient occurs is smaller than the relaxation length, the values of Seebeck and Peltier coefficients differ from those appropriate for a bulk conductor.

In a system comprising several contacts, or a network of contacts, new features can arise. If the size of the bulk region separating contacts is smaller than, or of the same order as, the inelastic mean free path, equilibrium in the bulk will not be established, which in turn results in new aspects of ballistic Peltier and Seebeck effects.

The above ballistic kinetic effects are potentially important and can manifest themselves in electronic microcircuits with a super-high degree of integration corresponding to a single-element size of the order of 10^{-5} cm, especially at temperatures below liquid-nitrogen temperature.

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